→ RIDDLE #3

ESA's NEO Coordination Centre

A visitor from outer space

Comet C/2020 F3 (NEOWISE) is a visitor that usually stays far away from the Sun. With an aphelion at about 700 au (more than 100 billion km!) it takes the comet more than 6000 years to complete an orbit around the Sun. At its perihelion it has an orbital velocity of 77.6 km/s, just 16 m/s too slow to escape from the solar system. But quite some dust that it ejects before or after its perihelion passage on 3 July 2020 will leave our solar system.

Figure 1: Comet C/2020 F3 (NEOWISE). Credit: R. Kresken

And here comes our next riddle:

- A comet with a semi-major axis of 700 au releases a jet of dust particles when reaching a distance to the Sun of 1.6 au.

  What is the size threshold where the dust particles will be ejected from the solar system because of the solar radiation pressure?

  Assume that the dust particles are ejected with zero velocity from the comet and ignore the gravity of the comet. Also assume perfectly absorbing surfaces and a specific density ρ of the particles of 1000 kg/m^3.

Please, send your responses before the proposed deadline to the following e-mail: neocc@ssa.esa.int.
Use as subject of your e-mail: "Riddle #3 – solution".
Moreover, please let us know if you would prefer not to have your name included in the list of correct replies.

European Space Agency

If you like to get some additional help, here is a spoiler (coded in the ROT 13/5 encryption commonly used in Geocaching):

Uvag: Gur fbyne enqvngvba cerffher npgf bccbfvgr gb gur Fha tenivgl. Guvf pna or rkcybvgrq gb erqhpr gur tenivgngvbany pbafgnag zh va gur ivf-ivin rdhngvba.

Decryption Key												
A	B	C	D	E	F	G	H	I	J	K	L	M
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
(letter above equals below, and vice versa)												

Answer

The semi-major axis of a dust particle can be computed by re-arranging the vis-viva equation:

$$a = \left(\frac{2}{r} - \frac{v^2}{\mu_{red}}\right)^{-1}$$

with $r = 1.6$ au, $v^2 = \mu(2/r - 1/a_{comet})$ and μ_{red} the reduced gravitational constant that takes into account the solar radiation pressure (SRP). For instance if SRP amounts to 10% of the gravity pull then μ_{red} simply is 0.9 μ.

The separation between bound and unbound orbit is where the semi-major axis a becomes infinite and hence:

$$\frac{2}{r} = \frac{v^2}{\mu_{red}} \tag{1}$$

Defining $\mu_{red} = \mu(1-\beta)$ where β is the ratio between SRP and gravitational force, equation 1 gives:

$$(1-\beta) = \frac{v^2}{\mu}\frac{r}{2} = 0.99886$$

and therefore β = 0.00114. Which size d is required to have β = 0.00114? We calculate the two forces at 1 au:

$$\text{SRP} = 4.5 \cdot 10^{-6}\pi\frac{d^2}{4}$$

($4.5 \cdot 10^{-6}$ N/m² is the solar radiation pressure for an absorbing surface. It can be derived from the solar flux of 1367 W/m² just by diving it by the speed of light c.)

$$\text{Gravity} = \frac{\mu \cdot \rho \cdot \pi \cdot \frac{d^3}{6}}{1\ \text{au}^2}$$

with $\rho \cdot \pi \cdot d^3/6$ the mass of the particle.

In order to have SRP/Gravity $= \beta$ we need $d = 1.5 \cdot 4.5 \cdot 10^{-6} \cdot 1\ \text{au}^2/(\beta \cdot \mu \cdot \rho)$ which gives $d = 996\ \mu$m or roughly 1 mm. Objects smaller than this size will be ejected from the solar system.

Correct responses

This time we had only one correct answer by:

- Fernando Virgilio Roig

Wolfgang Wittholt was very close, but released the dust particles at perihelion. Next month, the riddle will hopefully be a bit easier.

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