→ RIDDLE #2

ESA's NEO Coordination Centre

Asteroid Day Special

We are living in times where conspiracy theories are very fashionable. See how easy it is to come up with your own theory: there are extraterrestrials living on Planet X that has a period of 104.7 years around our Sun. Each time their planet passes through its aphelion, they throw a massive rock towards the Earth. So the Earth is hit by such a rock exactly every 104.7 years. When it happened the last but one time, it created quite a massive explosion in 1908 in the Tunguska region. We assume today that the asteroid had a size of about 40 m. And now we commemorate this event every year on 30 June as Asteroid Day.

The last event was the explosion of an asteroid of almost 20 m over Chelyabinsk on 15 February 2013 where about 1500 people went to hospital because they got hurt by the massive shock wave of the detonation of the asteroid right above their city.

Now we have to prepare for the next blow of the extraterrestrials, which should happen about 38216 days (104.7 years) later. Having a look at the Risk Page of ESA's Planetary Defence Office, we find that there are about 1063 objects listed that could hit the Earth in the next hundred years, i.e. about 0.2 impact risks per week. But have a look, in the week after the calculated impact date, there is a cluster of risk objects!

And here comes the riddle:

- Calculate the size of the asteroid that Planet X will throw at us and that will hit us in year 2222. To do this, assume that the mass of the rocks they throw towards the Earth is directly proportional to their economic power. Assume that the economic power follows the law $GNP = A \cdot t^2 + B \cdot t + C$. Derive the coefficients based on the data points you get from 1908, 2013 and 2117. (N.B.: for 2117, add up all possible impactors that are listed in the risk page within 7 days of the predicted return date).
- Bonus questions #1: what is the minimum travel time of the asteroids that will hit us in 2117?
- Bonus questions #2: assuming now that the rocks' travel time is always equal to Planet X's period, when did or will economic recession on Planet X end?

Please, send your responses before the proposed deadline to the following e-mail: neocc@ssa.esa.int.
Use as subject of your e-mail: "Riddle #2 – solution".
Moreover, please let us know if you would prefer not to have your name included in the list of correct replies.

Answer

We got a very nice answer from Tony Evans which we want to partly share. He wrote:

The legend of Planet X

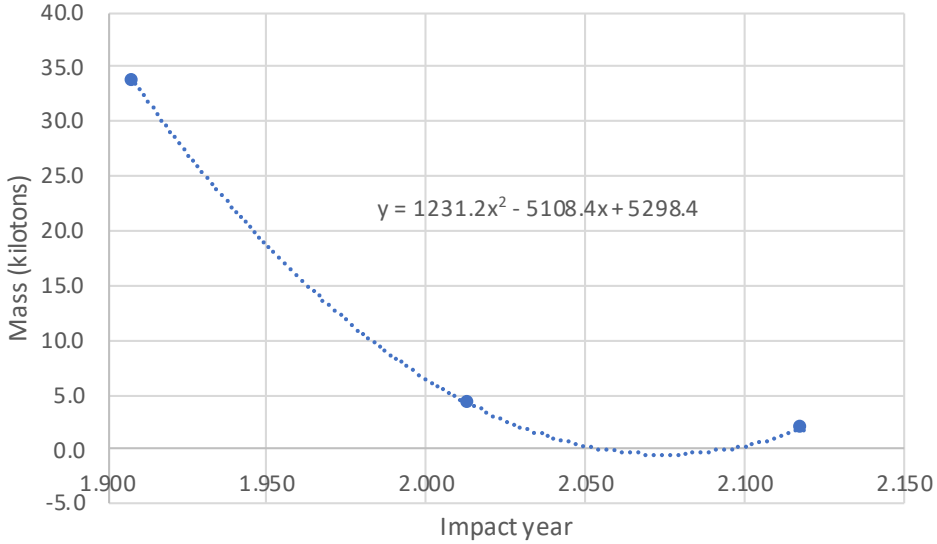
The inhabitants of the Planet X have a pretty miserable time. Their planet has an orbit taking 38216 days = 104.629 years to go round the Sun. This implies their orbit has a semi-major axis of 22.204 AU (Kepler 3rd law). That means they live out in the frozen wastes between Uranus and Neptune.
The High-priestess of Planet X decreed that the only way to salvation would be to exterminate the inhabitants of the planet Earth who seemed to have such a privileged place in the Goldirocks zone of the Solar System. Mythology told them that, long ago, a dominant species on Earth had been exterminated by the impact of a large rock. Maybe they could engineer something like that again and wipe out the humans.
All the resources of their civilisation were focussed on capturing and projecting a 40 m rock towards Earth. Eventually the rock struck the Earth and there were great celebrations.
Sadly, the impact seemed to have little effect on the humans. Few if any were killed and most of the population of Earth did not even realise anything had happened. They tried again one of their "years" later, but their civilisation was declining and they had resources only to project a 20 m rock. A few more humans were hurt or frightened this time but nothing serious to affect the population of Earth.
A third attempt was initiated. This time resources were even less and it was decided that it would be easier to project three smaller rocks rather than a single big one. By that time, the humans had become aware of dangerous rocks flying about and had established observation posts to track them.

We love this story from Tony. Very convincing, isn't it? So, let us first query the Risk Page of ESA's Planetary Defence Office for objects that could hit the Earth in early October 2117. And indeed, although the average is only 0.2 candidates per week, we find three:

Designator	Size (m)	Possible impact date (m)
2010 TW54	11	2117-Oct-09
2017 TA5	13	2117-Oct-08
2018 TT6	2.9	2117-Oct-06

Let us assume the density of these rocks is 1 g/cm^3 (this actually does not influence the results) and that they are perfect spheres with a diameter of 40 m for the 1908 event, 20 m for 2013 and the three diameters listed above for 2117.
This gives us the following impact masses: 33.5 kt for 1908 event, 4.2 kt for 2013 and 1.86 kt for 2117.
Fitting the 3 data points with a parabola, we get for instance the coefficients listed in the figure below (when dividing the year by 1000).
Extrapolating to the year 2222, the mass will be 26.5 kilotons (this depends on the chosen density) and the size is 37 m (no matter which density was chosen).

Bonus question #1: the 2117 impactor was detected close to Earth in 2010, and therefore if it will impact in 2117, it will be in orbit for at least 107 years. But before coming close to Earth in 2010 it must have travelled a time equivalent to at least half the orbital period of the transfer orbit.
Since Planet X takes 104.7 years to go around the Sun, then its orbit have a semi-axis of about 22 au. If the asteroid they sent us has aphelion at their distance, assuming it is in a circular orbit and has much smaller perihelion, then the *full axis* of the impactor orbit is about 22 au, implying a semi-axis of about 11 au, i.e. an orbital period of about 37 years.



Therefore, the minimum travel time is 18.5 plus 107 years.

Bonus question #2: since every good conspiracy theoretician ignores some basic facts, we ignore the answer to bonus question 1 and assume that the travel time is 104.7 years. The parabolic fit gives an economic minimum for an impact year of 2074.
Subtracting the 104.7 years leads us to a launch year of 1969, the year of the moon landing - which we all know, never happened. Another "proof" of our conspiracy theory. ☺

Correct responses

And this is the list of persons that have responded correctly to this riddle, in order of reception date:

- Kester Habermann
- Fernando Virgilio Roig
- Mauro Pirarba
- Tony Evans

Congratulations to all for the great work done and thank you so much for participating in this riddle.