→ RIDDLE #5

ESA's NEO Coordination Centre

Scanning the sky like a fly

The explosion of a 20 metre meteoroid over the city of Chelyabinsk happened just 7 years ago. Many buildings were damaged and more than 1500 people were hospitalised. And there was no pre-warning. In order to reduce the risk that we are taken by surprise by an asteroid that is impacting on Earth, ESA and NASA are regularly scanning the sky to detect these potentially hazardous objects early enough.

Now ESA wants to reinforce its efforts and develops a new telescope called the Flyeye Telescope. Why Flyeye? Because the incoming light is reflected from its 1.15 m main mirror to a beam splitter that distributes the field-of-view to 16 different cameras (see Fig. 1), just like the facets of an eye of a fly produce a compound image of its environment. With this novel approach, with one exposure an area in the sky measuring 6.7 deg by 6.7 deg can be imaged. Rather than scanning the full sky over two weeks, the Flyeye Telescope can scan the entire night sky in two nights. The final location of the first Flyeye Telescope will be Monte Mufara in Sicily (Italy) and a second telescope will go to Chile.

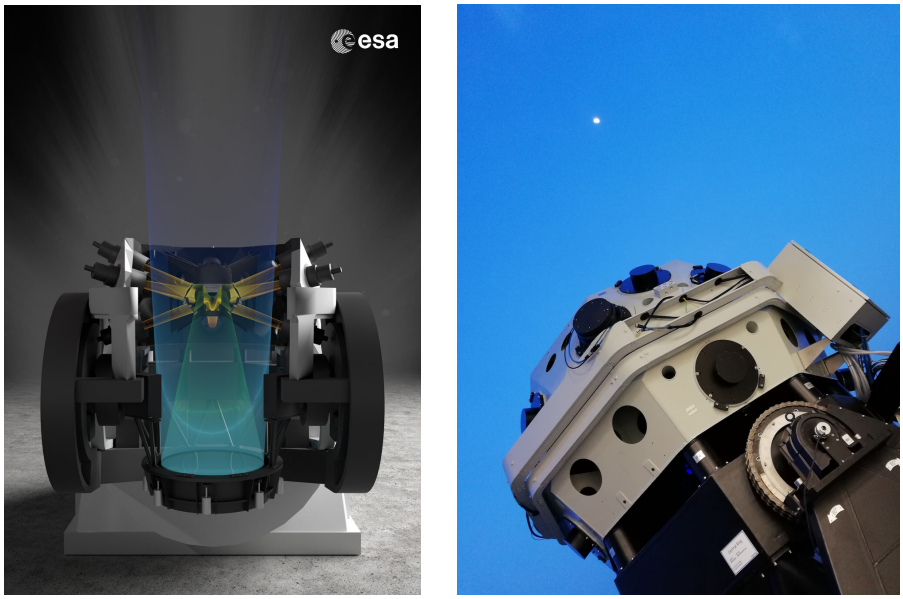

Figure 1: ESA's Flyeye telescope. On the left: light pathways through the telescope. On the right: the telescope watching the sky at its integration site, Turate (Italy)

In order to tell the telescope where to look at each moment of the night, an observation plan has to be established. To make life a bit easier, programmers usually divide the sky in fields (this is called tessellation of the sky) and write scripts that tell the telescope to move at time t to a field with the field-ID x. For the ESA telescope in Tenerife these fields are arranged in latitude bands. Of course as the latitude circles shrink towards the North pole, these fields start to overlap more and more, but not too much. For the Flyeye telescope we have one additional difficulty as the field-of-view of the telescope is rotated by 45 deg.

And here comes our next riddle:

Design a tessellation of the sky with squares measuring 6.7 deg by 6.7 deg and rotated by 45 deg such that the whole sky is covered, i.e. there must not be gaps between the squares. We are looking for the solution with the smallest number of squares but a complete coverage of the sky.

Answer

We admit that this was a tough riddle. And we received zero answers. Luca Conversi, responsible for all NEO observations at ESA, found a solution with 1095 fields. Whether this is the absolute minimum, we do not know, but the overlap is almost zero at low latitude and still very good at higher latitudes. In this solution the declination band between -55 and +55 degrees is covered with 23 rings of 39 rotated squares each. Then the latitude rings are reduced to 24 squares, finally at $\pm$80 degree latitude to 12 squares and finally there are 3 squares reaching the North pole (and symmetrically the South pole). Fig. 2 shows the proposed tessellation.

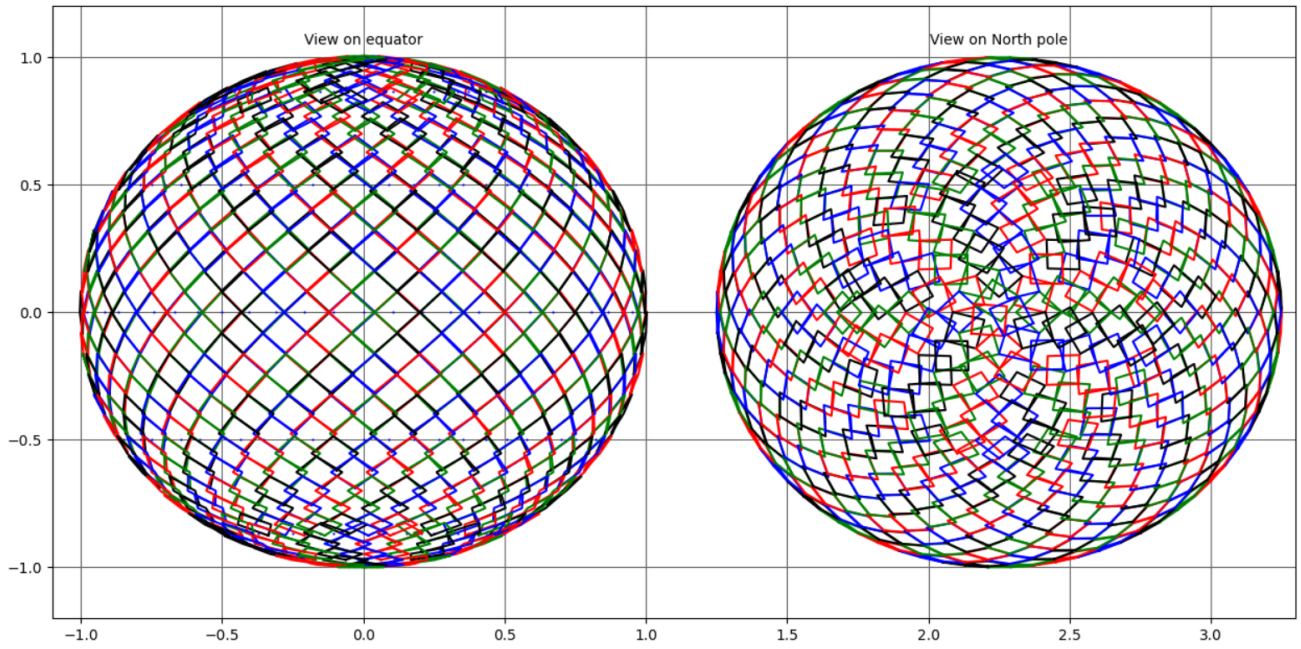

Figure 2: Flyeye tessellation of the sky with a total of 1095 fields